# Darcy Constant for Multisized Spheres with No Arbitrary Constant

**Robert T. Hamilton**
Weyerhaeuser Company, Tacoma, WA 98477

MacDonald et al. (1991) have presented a model for flow resistance in a bed of multisized spheres. Their derived Darcy permeability $k$ is a function of the first and second moments of the particle-size distributions

$$k = \frac{1}{180} \frac{\epsilon^3}{(1-\epsilon)^2} \left( \frac{M_2}{M_1} \right)^2 \tag{1}$$

where $\epsilon$ is the bed porosity and $M_1$ and $M_2$ are the first and second moments. The constant (1/180) was used as the best number to reconcile theory and experiment. Reasonable agreement between Eq. 1 and the experiment was found (see MacDonald et al., 1991, Figure 5).

We propose that the Darcy permeability $k$ for spheres can be derived from first principles and without recourse to arbitrary constants such as (1/180). Happel and Brenner (1973) have provided an expression for $k$ when flow is around spheres of radius $R$

$$k = R^2 \left( \frac{2}{9\gamma^3} \right) \left( \frac{3 - \frac{9}{2}\gamma + \frac{9}{2}\gamma^5 - 3\gamma^6}{3 + 2\gamma^5} \right) \tag{2}$$

Here $R$ is the sphere radius and $\gamma^3 = (1-\epsilon)$. We now show how this can be generalized to multisized systems.

For a multisized system, the $j$th moment can be written as

$$M_j = \sum_i D_i^j n_i \tag{3}$$

where $n_i$ is the number of particles with diameter $D_i$.

The total area ($S_T$) of a multisized system, given that the number and size of each particle fraction present is known, is given by

$$S_T = \sum_i n_i S_i = \sum_i n_i \pi D_i^2 \tag{4}$$

where $S_i$ is the area of a single sphere of diameter $i$.

In terms of the particle moments for a multisized system, a diameter $D_m$ can be defined as

$$D_m^2 = \frac{S_T}{\pi n_T} = \frac{\pi \sum_i n_i D_i^2}{\pi \sum_i n_i} = \frac{M_2}{M_0} \tag{5}$$

where $n_T$ is the total number of particles. For a multisized system, if the weight fraction $x_i$ of each size is known, the total mass of the system is $m_T$, and the particle density is $\rho_P$, then

$$x_i m_T = n_i \frac{\pi}{6} D_i^3 \rho_P \tag{6}$$

giving

$$n_i = \frac{x_i m_T}{\frac{\pi}{6} D_i^3 \rho_P} \tag{7}$$

Substituting Eq. 7 into Eq. 5 and canceling the terms gives

$$D_m^2 = \frac{\sum_i \frac{x_i}{D_i}}{\sum_i \frac{x_i}{D_i^3}} \tag{8}$$

Replacing $D_m^2$ by $4R_m^2$, Eq. 8 can then be substituted into Eq. 2 to give (substituting $(1-\epsilon)$ for $\gamma^3$ at the same time)

$$k = \left( \frac{1}{12} \right) \left( \frac{3(1-\epsilon)^{5/3} - 3(1-\epsilon)^{1/3} - 2(1-\epsilon)^2 + 2}{(1-\epsilon)[2(1-\epsilon)^{5/3} + 3]} \right) \times \left( \frac{\sum_i \frac{x_i}{D_i}}{\sum_i \frac{x_i}{D_i^3}} \right) \tag{9}$$

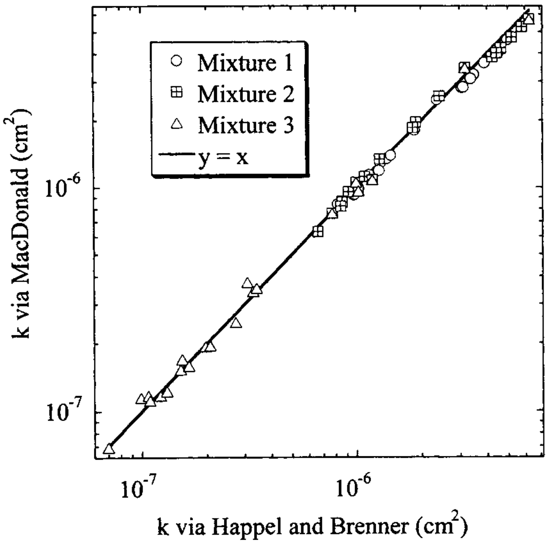


**Figure 1. Comparison of *k* derived: MacDonald (1991, Eq. 1) vs. theory of Happel and Brenner (1973, Eq. 9).**

Values for $\epsilon$, $D_i$ and $x_i$ were obtained from the three mixtures used by MacDonald.

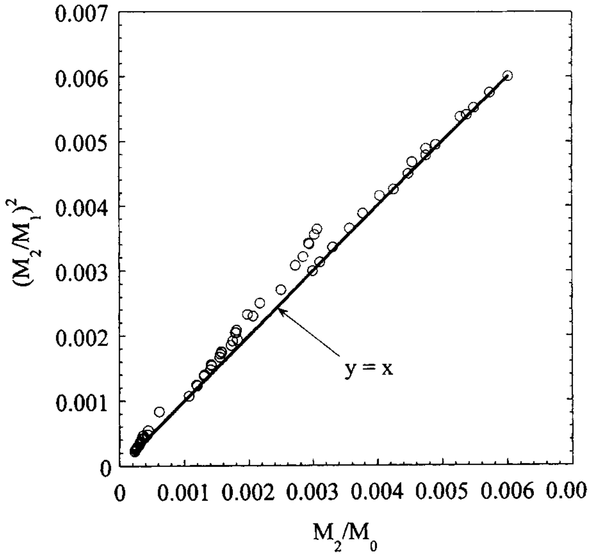


**Figure 2. Comparison of the square of the mean particle diameter calculated from the particle size distribution moments according to Eq. 1, $(M_2/M_1)^2$, and Eq. 5, $M_2/M_0$.**

Equation 9 contains no arbitrary constants. Figure 1 shows a comparison between the MacDonald $k$ values (Eq. 1) and Eq. 9 for the three mixtures used in the MacDonald study. The two results are nearly identical.

The close agreement between Eqs. 1 and 9 for the system in question is not surprising when the relationships are examined more closely. Each equation has a term that is a function of the total porosity, and a term that essentially calculates an effective particle radius based on the particle-size distribution. Figure 2 compares the result of the effective particle radius for the two equations. For the systems studied, the two methods are nearly identical (average error of 8%) for almost all of the mixtures.

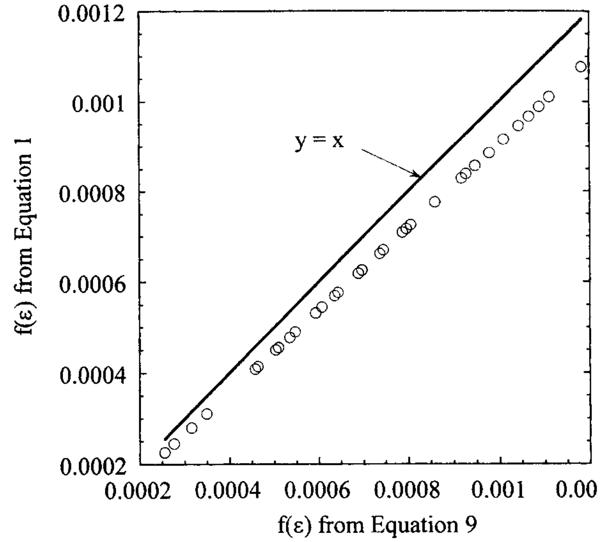


**Figure 3. Comparison of the porosity term $f(\epsilon)$ from Eq. 1, $\left(\frac{1}{180}\,\frac{\epsilon^3}{(1-\epsilon)^2}\right)$, and Eq. 9, $\left(\frac{1}{12}\right)$ $\times\left(\frac{3(1-\epsilon)^{5/3}-3(1-\epsilon)^{1/3}-2(1-\epsilon)^2+2}{(1-\epsilon)[2(1-\epsilon)^{5/3}+3]}\right)$.**

The porosity terms in the two equations are compared in Figure 3. Again, agreement between the relatively simple relation of Eq. 1 and the more complicated relation of Eq. 9 is good. In fact, agreement would be almost perfect if the constant in Eq. 1 were (1/162) instead of (1/180).

Equation 9 thus provides a direct and unambiguous method for determining the resistance to flow of a mixture of spheres. No arbitrary constant is needed.


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## Literature Cited


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